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# THE SUBSTITUTIVE CIRCLES FAMILY METHOD – GRAPHICAL APROACH IN CATIA DESIGN ENVIRONMENT

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## ABSTRACT

*The profiling of tools which generate by enveloping using the rolling method is done using the general principles of surfaces enveloping – first Olivier theorem or Gohman method. Also, complementary methods for the study of the enveloping surfaces, associated with a pair of rolling centrodes were elaborated: "the minimum distance" method; "the substitutive circles family" method; "the in-plane generating trajectories" method. In this paper, a graphical solution of rack-gear and gear shaped tool profiling is proposed. The solving method is based on the CATIA design environment capabilities. The method is simple, intuitive and rigorous. An application example of the method is presented versus an analytical method.*

**KEYWORDS:** surface enveloping, tool's profiling, CATIA

## 1. INTRODUCTION

The general theory of the enveloping surfaces is based on the Olivier fundamental theorem of enveloping surfaces [1] or in the Gohman variant [1], [2], which, as is known have the capability to allow the solving of the all issues linked with: tools profiling, which generate ordered curls of profiles (rack gear and gear shaped tools); profiling of tools for generating of helical surfaces (revolution tools; cylindrical tool and helical tools); tools which generate by the single point contact, secondary order degree contact (worm mill generating an ordered profiles curl associated with one axode).

In various situations, the application of the fundamental theorems leads to analytical formulation of the complicated problems by complex equations of which handle may be difficult.

Were developed complementary theorems of the surfaces enwrapping [3], specific for the characteristic modes of various machining types: generating of surfaces ordered curls associated with a pair of rolling centrode or profiling of hob mill which generate surfaces by single point contact. For these cases were states specifically theorems: the substitutive circle's theorem; the minimum distance theorem or the in-plane generating trajectories theorem [4].

In the same time, specific algorithms were developed, based on analytical development of these complementary methods and were made applications for specific domains as profiling of the hob mill for generating helical surfaces.

Also, are known recently developments in the CATIA graphical design, by interpretation of the fundamental enveloping theorem and elaboration of specific algorithms in VBA as 3D applications for generating tools profiling [6], [7], [8], with rack-gear tools, hob mill, tools for helical surfaces generation. In this paper is proposed a graphical solution, developed in CATIA, based on the complementary theorem of the substitutive circles family for rack-gear tool.

## 2. THE THEOREM OF SUBSTITUTIVE CIRCLES FAMILY

For a profile belongs to an ordered curl of profiles and associated with an axode (centrode) is possible to define a family of circles, tangents with the given profile and with centres on the associated centrode – the "substitutive circles" family, see figure 1.

The current circle of substitutive circles family, in the $XY$ reference system associated with $C_1$ centrode; have in the $O_i$ point with coordinates:

$$x_{O_i} = -R_{rp} \cdot cos\,\beta_i\,;\; y_{O_i} = R_{rp} \cdot sin\,\beta_i\,, \tag{1}$$

with $\beta_i$ arbitrary variable.

In this way, the current circle parametrical equations may be written in form:

$$C_i \left| \begin{array}{l} X = -R_{rp} \cdot cos\,\beta_i + r_i \cdot cos\left(\beta_i + \theta\right); \\ Y = R_{rp} \cdot sin\,\beta_i + r_i \cdot sin\left(\beta_i + \theta\right). \end{array} \right. \tag{2}$$

with $\theta$ variable parameter.

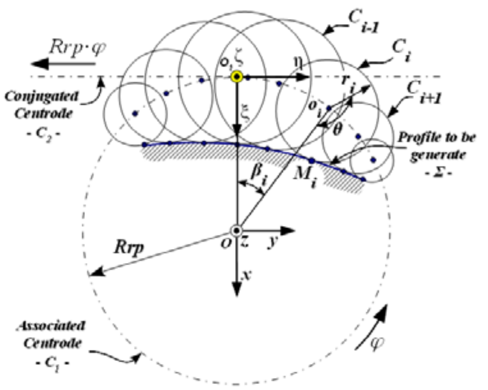


Fig. 1. The family of substitutive circles, reference systems

Knowing the equations of the $\Sigma$ profile with $u$ variable parameter:

$$\Sigma \left| \begin{array}{l} X = X\left(u\right); \\ Y = Y\left(u\right). \end{array} \right. \tag{3}$$

From the equations:

$$\begin{array}{l} X\left(u\right) = -R_{rp} \cdot cos\,\beta_i + r_i \cdot cos\left(\beta_i + \theta\right); \\ Y\left(u\right) = R_{rp} \cdot sin\,\beta_i + r_i \cdot sin\left(\beta_i + \theta\right); \\ \dot{X}_u = -r_i \cdot sin\left(\beta_i + \theta\right); \dot{Y}_u = r_i \cdot cos\left(\beta_i + \theta\right), \end{array} \tag{4}$$

it is possible to find the value of the $r_i$ radius of the substitutive circle and the value of the variable parameter $\theta$, corresponding with the tangency point with the $\Sigma$ profile – the $M_i$ point, see figure 1.

The specifically theorem of the substitutive family circles [3] may be states: The enveloping of the $\Sigma$ profile, in the rolling process of the $C_1$ and $C_2$ centrodes pair, is the enveloping of the substitutive circles family, in the rolling process, onto the associated centrode.

For rack-gear generation, the relative motion is defined as:

$$\xi = \omega_3^T\left(\varphi\right) \cdot X - \left| -R_{rp} \quad -R_{rp} \cdot \varphi \right|^T, \tag{5}$$

with $\varphi$ motion variable parameter.

Considering equations (4), from (5) it is deduced the substitutive circles family in the rack-gear reference system, in form:

$$C_i\left(\varphi\right) \left| \begin{array}{l} \xi = \xi\left(u,\varphi\right), \\ \eta = \eta\left(u,\varphi\right). \end{array} \right. \tag{6}$$

The enveloping of the $(C_i)_\varphi$ circles family represents the rack-gear tool’s profile.

## 3. “THE SUBSTITUTIVE CIRCLES FAMILY” METHOD DEVELOPED IN CATIA

It is proposed the development of “substitutive circles family” method using the capabilities of the CATIA graphical design environment.

For a profile belongs to an ordered profiles curl associated with a circular centrode with $R_{rp}$ radius. The value of this radius meet the condition that the normal, in every point belongs to profile, to intersect the centrode with $R_{rp}$ radius, is determined the substitutive circles family, according with previous definition.

It is determined the arc of the $R_{rp}$ circle, which correspond to the analysed profile, $\overset{\frown}{AB}$, see figure 2.

The circle arc with $R_{rp}$ radius is divided in a number of equal segments:

$$\beta = \overset{\frown}{AB}/n\,. \tag{7}$$

In every point $O_i$, see figure, from the circle with $R_{rp}$ radius is described the normals to $\Sigma$ profile, generating in this way the substitutive circles family.

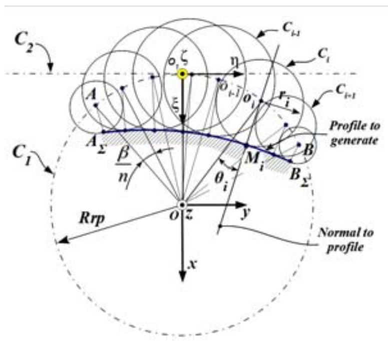


Fig. 2. The family of substitutive circles– the circles family associated with Σ profile

The $\overset{\frown}{O_{i-1}O_i}$ arc’s length, in the rolling process of the two centrodes, is transposed onto the rack-gear centrode, see figure 3.

The circles belongs to the substitutive family are draw with centers onto the $C_2$ centrode, representing the transpose of the circles family $(C_i)_\beta$.

The enveloping of the circles family $(C_i)_\beta$ represents the $S$ profile of the rack-gear tool.

It is possible to determine the contact points between the family circles and the $S$ rack-gear's profile.

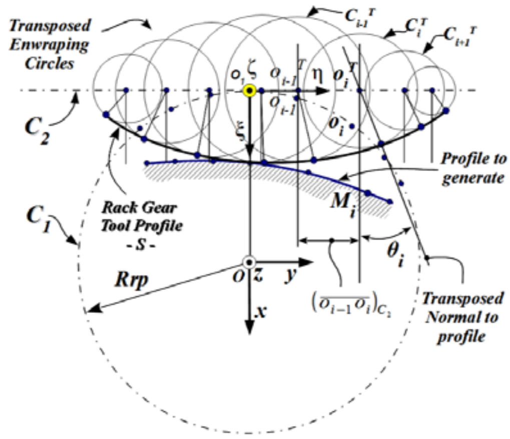


Fig. 3. The family of substitutive circles, transposed onto the centrode associated with rack-gear

Firstly, in the CATIA design environment – Generative Shape Design, it is drawn the profile composed from two circle arcs $\widehat{AB}$ and $\widehat{BC}$, the rolling circle with $R_{rp}$ radius and the gearing curve. In order to find the whole tool's profile there are drawn normals to the arcs $\widehat{AB}$ and $\widehat{BC}$, in their end points, $A$ and $C$, using the *Projection* command. The intersection points between these normals and the rolling circle with $R_{rp}$ radius, separate on this an arc from the rolling circle, $\widehat{O_A O_C}$, afferent to the $ABC$ profile. Onto this arc are selected $n$ equidistant points, dividing the $\beta$ angle into equal angles with value $\beta/n$, see figure 4.

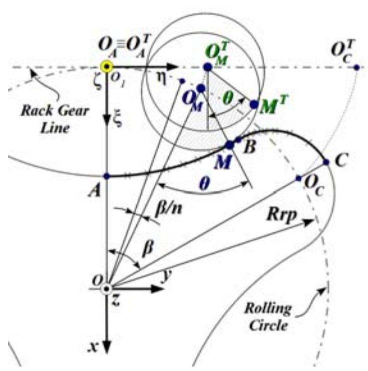


Fig. 4. The points selection onto the *ABC* profile

## 4. APPLICATIONS

An application of the graphical method is presented, versus an analytical method for the profiling of the rack-gear tool. The profile is composed from circle arc, see figure 5.

The rolling circle with $R_{rp}$ radius is defined (the centrode associated with the curl of profiles to be generated), see figure 5 and, also, the substitutive circles family for the two constructive arcs of the rotor's profile $\widehat{ABC}$.

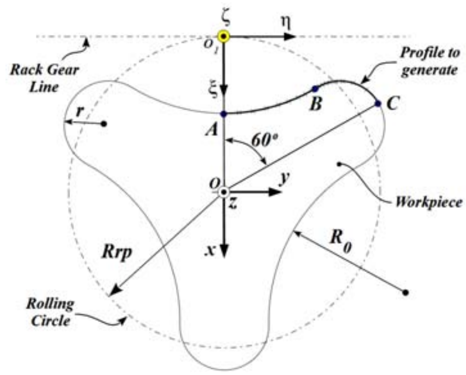


Fig. 5. The rotor to be generated and the its centrode

A profile with $R_0 = 40$ mm; $R_{rp} = 35$ mm; $a = 22.72$ mm; $R = 30$ mm, $r = 10$ mm, is considered for generation.

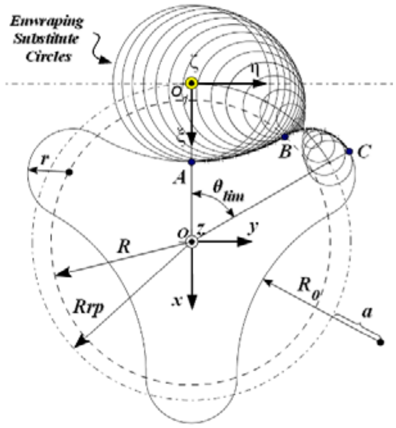


Fig. 6. The family of substitutive circles

The substitutive circle's family is made for an angular pitch,

$$\beta = \theta_{lim}/N \text{, } N \text{ positive integer value.} \quad (8)$$

The $\theta_{lim}$ angle is determined from the condition of circle's intersection,

$$\begin{array}{c|c} \Sigma & \left(X - R_{rp} - a\right)^2 + Y^2 = R_0^2, \\ C_1 & X^2 + Y^2 = R_{rp}^2. \end{array} \quad (9)$$

The substitutive circle's family has the centers at pitch, $p = R_{rp} \cdot \beta$, see figure 7.

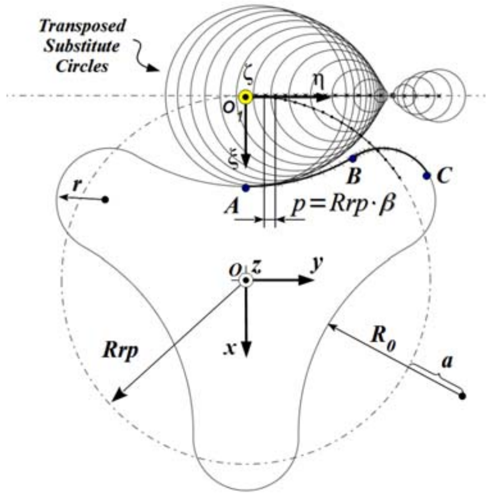


Fig. 7. The family of substitutive circles

The enveloping of the circles family represents the rack-gear tool's profile, *S*.

For the same profile it was calculated the rack-gear tool's profile using the in-plane trajectories method.

In order to compare the two tools profiles it was use the fitting curves toolbox of the MatLab program. Subsequently, analyzes of the two fittings was done for a vector corresponding to the $\eta$ axis. The relative differences between the functions values are presented in table 1.

The relative difference between the $\xi_C$ and $\xi_T$ calculated with the substitutive circles family method and in-plane trajectories method is give by:

$$\Delta = \left| \frac{\eta_C - \eta_T}{\eta_C} \right| \cdot 100 \,. \tag{10}$$

The form of the rack-gear tool's profile is presented in figure 8.

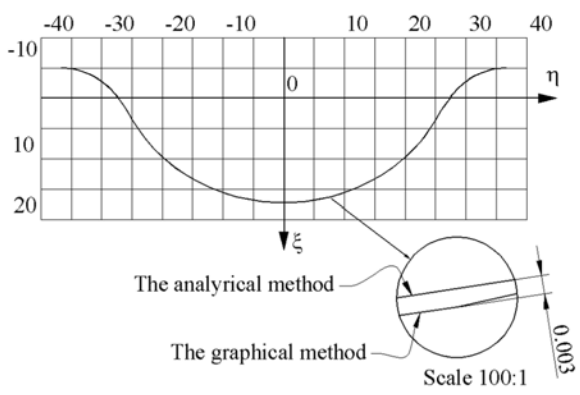


Fig. 8. The secondary order tool's profile

Table 1. The rack-gear tool's profile and relative difference ($\xi_C$ – coordinate determined in CATIA, $\xi_T$ – coordinate calculated analytical)

| Crt. no. | $\xi_C$ [mm] | $\xi_T$ [mm] | $\eta$[mm] | $\Delta$ [%] |
|---|---|---|---|---|
| **1.** | -4.971 | -4.977 | -36.000 | 0.127 |
| **2.** | -4.139 | -4.135 | -32.400 | 0.097 |
| **3.** | 8.169 | 8.178 | -21.600 | 0.100 |
| **4.** | 11.419 | 11.412 | -18.000 | 0.061 |
| **5.** | 13.702 | 13.701 | -14.400 | 0.007 |
| **6.** | 15.332 | 15.331 | -10.800 | 0.007 |
| **7.** | 16.432 | 16.432 | -7.200 | 0.000 |
| **8.** | 17.071 | 17.070 | -3.600 | 0.006 |
| **9.** | 17.280 | 17.280 | 0.000 | 0.000 |
| **10.** | 17.071 | 17.070 | 3.600 | 0.006 |
| **11.** | 16.432 | 16.432 | 7.200 | 0.000 |
| **12.** | 15.332 | 15.331 | 10.800 | 0.007 |
| **13.** | 13.702 | 13.701 | 14.400 | 0.007 |
| **14.** | 11.419 | 11.412 | 18.000 | 0.061 |
| **15.** | 8.169 | 8.177 | 21.600 | 0.097 |
| **16.** | -4.139 | -4.135 | 32.400 | 0.097 |
| **17.** | -4.971 | -4.977 | 36.000 | 0.127 |

## 5. CONCLUSIONS

The "substitutive circles family" method allows the solving of the issue of determination the rack-gear tool for generation of an ordered profiles curl.

The same problem solving by CATIA graphical method is a rigorous solution. The method allows the easy determination of rack-gear profile for simple blank profiles. The method may be extended also for complex profiles.

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